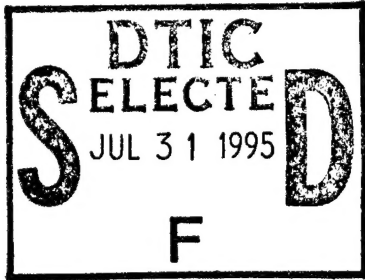# Point Formation and Related Agreement Problems for Synchronous Mobile Robots with Limited Visibility*

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March 1, 1995

Abstract   We discuss fundamental formation and agreement problems for autonomous, synchronous robots with limited visibility. Each robots is a mobile processor that, at each discrete time instant, observes the relative positions of those robots that are within distance V of itself, computes its new position using the given algorithm, and then moves to that position. The main difference between this work and many of the previous ones is that, here, the visibility of the robots is assumed to be limited to within distance V, for some constant $V > 0$. The problems we discuss include the formation of a single point by the robots and agreement on a common x-y coordinate system and the initial distribution, and we present algorithms for these problems, except for the problem of agreement on direction (a subproblem of agreement on a coordinate system), which is not solvable even for robots with unlimited visibility. The discussions we present indicate that the correctness proofs of the algorithms for robots with limited visibility can be considerably more complex than
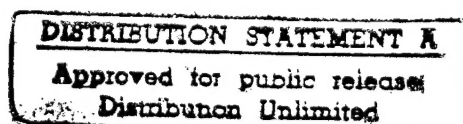
those for robots with unlimited visibility.

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1 Introduction

Suppose that a large group of soldiers are scattered in a foggy battlefield, where visibility is limited to only, say, 20 meters. For instance, a soldier may (faintly) see three other soldiers, but he might lose sight of them if he moves even slightly. Under such a circumstance, is it possible for the soldiers to gather, silently, at a single location? We answer this and related questions using a formal model of distributed autonomous mobile robots with limited visibility.

In recent years, interest in distributed autonomous robot systems has increased considerably. Leading research activities in the area include the Cellular Robotic System (CEBOT) of Fukuda *et al.* [6] [8] [16], the Swarm Intelligence of Beni *et al.* [1] [2], the Self-Assembling Machine ("fractum") of Murata *et al.* [10], experimental and theoretical investigations on formation and agreement problems for anonymous mobile robots by the authors *et al.* [12] [13] [14] [15], and others [3] [5] [7] [9] [11] [17]. One of the main issues in these works is the study of cooperative behavior of autonomous robots that operate under distributed control. Distributed control means that each robot makes its own decisions using the given algorithm based on the available information. No global controller is assumed to exist.

In this paper, we continue the authors' previous works on formation and agreement problems for anonymous mobile robots [12] [13] [14] [15]. The goal of a formation problem is to let the robots form, in finite steps, a given geometric figure or distribution, starting from an arbitrary initial distribution, using distributed control in which each robot has to make its own decision in each step based on the behavior of other robots that it has observed. Heuristic algorithms for forming an approximation of a circle, a line segment, and a simple polygon have been proposed in [12] [15]. Formal discussions on the power and limitation of the distributed method for formation problems can be found in [13] [14].

In an agreement problem, on the other hand, the robots are required to reach, in finite steps, a state in which they all have a common understanding of the given concept, such as a location at which they gather, direction in which they move, and distance over which they move. Leader election can also be considered as an agreement problem. The problem of agreeing on a common x-y coordinate system, which is perhaps the most fundamental agreement problem when initially the robots do not have a common coordinate system, is discussed in [13] [14].

In many of the works mentioned above, including the authors' previous works, it is assumed that the sensor range of a robot is unlimited, that is, a robot is capable of seeing other robots regardless of the distance to them. (Exceptions include the works that discuss collision avoidance strategies that use only local information.) In this paper, we assume that each robot has only *limited visibility*, in the sense that it can see and know the relative positions of only those robots that are within distance V of itself, for some constant $V > 0$. V therefore represents the visibility range of the robots. Under this assumption, we discuss fundamental formation and agreement problems, namely, the formation of a single point by the robots, agreement on a common x-y coordinate system (i.e., agreement on the origin, unit distance, and direction of the positive x-axis), and discovery of and agreement on the initial distribution. The goal of the single point formation problem is to move the robots to a single point in finite steps. And, as we stated above, agreement means that the robots must obtain a common understanding of the given concept in finite steps. We present algorithm for solving these problems, except for the problem of agreement on direction (a

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subproblem of agreement on a coordinate system), which is not solvable even for robots with unlimited visibility.

In fact, a close relationship exists between formation problems and the problem of agreeing on a common x-y coordinate system. The problem of agreeing on the origin, which is reducible to the problem of agreeing on a point, is further reducible to the problem of forming a point. The problem of agreeing both on the origin and on unit distance is reducible to the problem of forming a circle. Also, the problem of agreeing on direction is reducible to the problem of forming a line segment. This observation partly motivates the study of the single point formation problem. (See [13] [14] for more details.)

The algorithm we present for the single point formation problem is "oblivious", in the sense that the position of a robot at the next time instant is determined based only on the positions of those robots that are *currently* visible, and independently of the past robot behavior. Hence, the algorithm can easily be implemented, since there is no need to store and process the history of robot moves.

More **importantly, since** real sensors and controllers are not always accurate enough, practical algorithms must take into account the effects of sensor and control errors, and tolerate them. The fact that our algorithm is oblivious and works correctly starting from any initial distribution (as is proved later), implies that the algorithm is extremely robust against sensor and control errors of the robots, in the following sense: The algorithm works correctly even in the presence of a finite number of (i.e., transient) such errors, since the robot distribution immediately after the occurrence of the last error can be viewed as a new initial distribution. The algorithm for agreement on a common x-y coordinate system is also oblivious, but the one for discovering and agreeing on the initial distribution is not.

It turns out that algorithms for robots with limited visibility can be considerably more complex than those for robots with unlimited visibility that solve the same problem. Similarly, proving the correctness of such algorithms for robots with limited visibility can be much more involved compared with the proofs for the case of unlimited visibility. This is mainly due to the fact that, under limited visibility, the behavior of a robot is based only on local information available to that robot, whereas the correctness of the algorithm can only be derived from the global behavior of the entire set of robots.

The model of the robot system we use is basically the same as that given in [13] [14], except that the robots have only limited visibility. Namely, each robots is a mobile processor that repeatedly does the following: Observe the relative positions of those robots that are within distance V of itself, compute its next position using the given algorithm, and then move to that position. The algorithm can use, as input, the positions of other robots observed by the robot in the past. We assume the following: (1) Initially, the robots do not have a common x-y coordinate system. (2) Initially, the robots do not have a sense of direction. (3) The robots are indistinguishable by their appearances. (4) All robots execute the same algorithm for determining their movement.

In this paper, we only consider the case when the robots are *synchronous*, that is, they always observe other robots and move simultaneously at discrete time instants $0, 1, 2, \ldots$. It has been reported in [13] [14] that certain formation and agreement problems can be extremely hard (or even unsolvable) if the robots are not guaranteed to be synchronous. The case of asynchronous robots is left for future research.

Taking into consideration collision avoidance of robots with volume is of course important, but for simplicity, in this paper we represent a robot as a point, and assume that two

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or more robots can occupy the same position simultaneously and the robots do not block the views of others.

We introduce the problem of forming a single point by the robots in Section 2, and present an algorithm for solving it in Section 3. The correctness of the algorithm is shown in Section 4. Section 5 discusses agreement problems. Concluding remarks are found in Section 6.

2 The Single Point Formation Problem

Let $R = \{r_1, \ldots r_n\}$ be the set of robots. We denote by $r_i(t)$ the position of robot r_i (in the 2-dimensional Euclidean space) immediately before the move at time instant t. $r_i(t)$ is called the position of r_i at t. The multiset $P(t) = \{r_1(t), \ldots, r_n(t)\}$ then denotes the distribution of the robots at t. ($P(t)$ is a multiset, since it is possible that $r_i(t) = r_j(t)$ for some $i \neq j$.) So $P(0)$ denotes the initial positions of the robots. Given $P(t)$, define a graph $G_t = (R, E_t)$, called the *visibility graph* at time t, by $(r_i, r_j) \in E_t \leftrightarrow dist(r_i(t), r_j(t)) \leq V$, where $dist(p, q)$ denotes the Euclidean distance between points p and q. That is, there exists an edge between r_i and r_j in G_t if and only if r_i and r_j are mutually visible at t. See Figure 1.

For convenience, we introduce the following notation. $S_i(t)$ denotes the set of robots that are visible from r_i at t, that is, $S_i(t) = \{r_j | dist(r_i(t), r_j(t)) \leq V\} \subseteq R$. Note that $r_i \in S_i(t)$. We denote by $C_i(t)$ the smallest enclosing circle of the set $\{r_j(t) | r_j \in S_i(t)\}$ of the positions of the robots in $S_i(t)$ at t, and $c_i(t)$ its center. Clearly, for any set S of points, the smallest enclosing circle of S is unique and is effectively computable [4]. The following property is well known [4]. The proof is omitted.

Proposition 1 *Let C be the smallest enclosing circle of a set S of points. Then either*

1. *there are two points p, q in S on the circumference of C such that the line segment $\overline{pq}$ is a diameter of C, or*

2. *there are three robots p, q, r in S on the circumference of C such that the center c of C is inside $\triangle pqr$.*

The *single point formation problem* is the problem of moving the robots in the same connected component of G_0 to a single point in finite steps, where G_0 is the visibility graph at time 0. Our goal is to design an algorithm for the robots that achieves this, regardless of the initial distribution $P(0)$. (The single point *convergence problem* discussed in [13] [14] only requires that the robots converge to a single point. The process of convergence need not terminate in finite steps.)

Note that two robots that belong to different connected components of G_0 need not move to the same point. In fact, under limited visibility, there is no deterministic algorithm for moving *all* robots to a single point. To see this, suppose that there are only two robots r_1 and r_2, such that (1) the local coordinate system of r_1 is obtained from that of r_2 by a translation of distance d, for some $d > V$, and (2) initially, r_1 and r_2 are at the origin of their respective local coordinate systems. Then initially, neither r_1 nor r_2 sees any other robot, and the situation looks identical to both. So if the algorithm they use is deterministic, they move (simultaneously) in the same manner using their respective local coordinate systems.
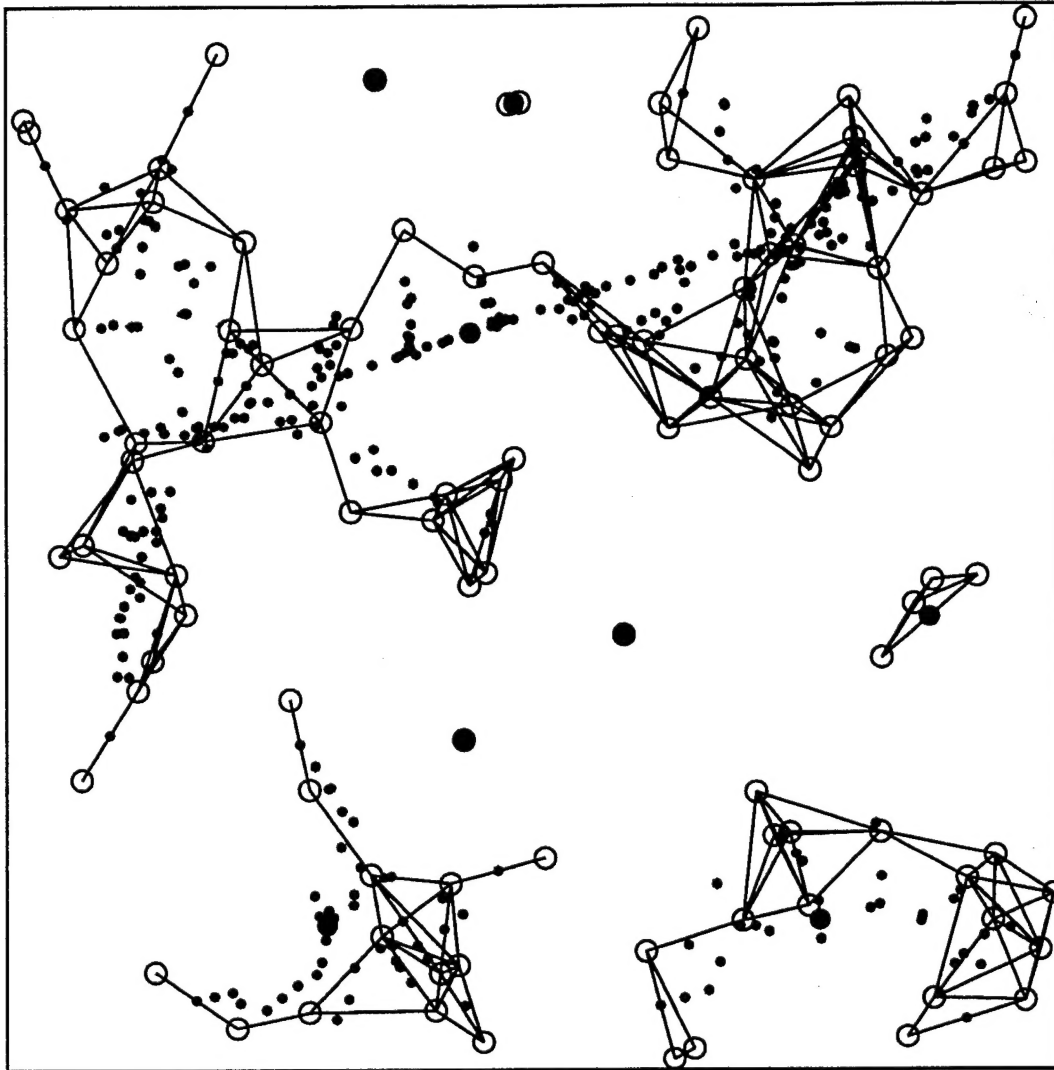
Figure 1: Hollow circles are the initial positions of 100 robots. Visibility graph G_0 consists of these circles and the edges among them. Solid circles are their final positions after the execution of the algorithm for the single point formation problem given in Section 3. Small dots represent their intermediate positions.
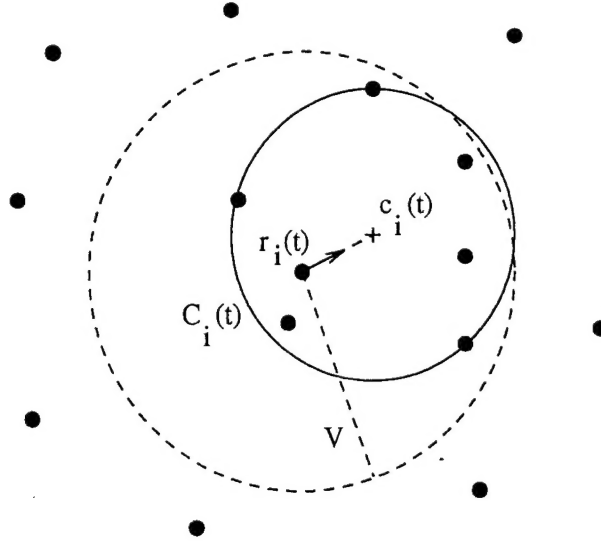
Figure 2: Direction of r_i's move.

This means, by the assumption on their coordinate systems, that the robots are again distance d apart and the situation looks identical to both. This argument continues, and thus the robots can never converge to a single point.

3   Algorithm

We present an algorithm for solving the single point formation problem. Intuitively, the algorithm solves the problem by achieving the following two subgoals at every time instant t: (1) The robots in the same connected component of G_t "get closer" in some sense at $t + 1$, and (2) robots that are mutually visible at t remain mutually visible at $t + 1$.

First of all, at every time instant t, if r_i does not see any robot other than itself (i.e., $S_i(t) = \{r_i\}$), then r_i does not move at t. Otherwise (i.e., $S_i(t) \supset \{r_i\}$), to achieve the first subgoal, we move r_i towards the center of the smallest enclosing circle of the positions of all the robots that r_i can see. Formally, at t, r_i moves towards the center $c_i(t)$ of $C_i(t)$, over some distance $MOVE$ to be specified below. See Figure 2.

If r_i moves at t as mentioned above, then we achieve the second subgoal as follows. Let r_j, $i \neq j$, be one of the robots in $S_i(t)$, that is, r_j is visible from r_i at t. Let m_j be the midpoint of $r_i(t)$ and $r_j(t)$. As is shown in Figure 3, if the next positions of r_i and r_j are both inside the disc D_j with center m_j and radius $V/2$, then r_i and r_j can still see each other at $t + 1$. Formally, given the direction of the move (towards $c_i(t)$, as explained above), r_i computes the maximum distance ℓ_j that it can move in that direction without leaving D_j, as follows. If $dist(r_i(t), r_j(t)) = 0$, then clearly $\ell_j = V/2$. Otherwise, let $d_j = dist(r_i(t), r_j(t))$ be the distance between r_i and r_j at t, and $\theta_j = \angle c_i(t) r_i(t) r_j(t)$ the direction of the move of r_i with respect to the ray from r_i to r_j, where $0 \leq \theta_j \leq \pi$. See Figure 4. Then

$$\ell_j = (d_j/2) \cos \theta_j + \sqrt{(V/2)^2 - ((d_j/2) \sin \theta_j)^2}.$$
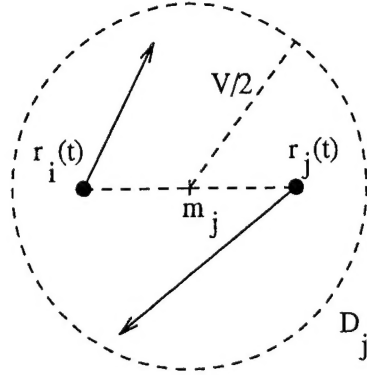
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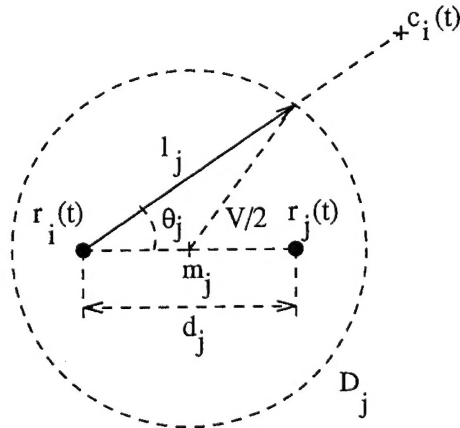Figure 3: Robots r_i and r_j remain mutually visible.

Figure 4: The maximum distance ℓ_j that r_i can move towards $c_i(t)$ without leaving D_j.

Robot r_i computes this ℓ_j for each $r_j \in S_i(t)$, and then finds

$$LIMIT = \min_{r_j \in S_i(t) - \{r_i\}} \{\ell_j\}$$

as well as

$$GOAL = dist(r_i(t), c_i(t)),$$

which is the distance from r_i to $c_i(t)$ at t. Finally, r_i moves over distance

$$MOVE = \min\{GOAL, LIMIT\}$$

towards $c_i(t)$. By the definition of $LIMIT$, r_i remains inside the disc D_j for every $r_j \in S$ after the move. Since all robots compute their next positions using the same algorithm, any pair of robots that are mutually visible at t remain mutually visible at $t + 1$.

4 Correctness

In this section, we prove that the algorithm given in Section 3 solves the single point formation problem. First, Lemma 1 states formally that robots that are mutually visible remain mutually visible during the execution of the algorithm.

Lemma 1 *For any two robots r_i, r_j and any time instant $t \geq 0$, $(r_i, r_j) \in E_t \rightarrow (r_i, r_j) \in E_{t+1}$.*

Proof The lemma follows from the definition of $LIMIT$ and the explanation given in the previous section. □

By Lemma 1, for any time instant $t \geq 0$, the robots in a connected component of G_t belong to the **same** connected component of G_{t+1}. Also, since there are only a finite number of robots, the number of times that different connected components merge is finite. Thus in the following, let t_0 be the smallest time instant such that no two connected components merge after t_0. Fix a connected component (S, A) of G_{t_0}, and for each $t \geq t_0$, let $CH(t)$ be the convex hull of the positions of the robots in S at t, that is, $CH(t)$ is the convex hull of the set of points $\{r_j(t) | r_j \in S\}$.

Lemma 2 states that the diameter of $CH(t)$ never increases, and Lemma 3 states that once the robots in a connected component gets sufficiently close to each other, then they move to the same position in one step.

Lemma 2 *For any $t \geq t_0$, $CH(t + 1) \subseteq CH(t)$.*

Proof Fix a robot $r_i \in S$. By the definition of $C_i(t)$ and Proposition 1, the center $c_i(t)$ of $C_i(t)$ is in the convex hull of the positions of the robots in $S_i(t)$ at t. Since the current position $r_i(t)$ of r_i is also in the same convex hull, so is the next position $r_i(t + 1)$ of r_i. But this convex hull is contained in $CH(t)$, since $S_i(t) \subseteq S$. So $r_i(t + 1) \in CH(t)$. Since this is true for any robot in S, $CH(t + 1) \subseteq CH(t)$ holds. □

Lemma 3 *If the diameter of $CH(t)$ is no greater than V, then all the robots in S move to the same point at t.*
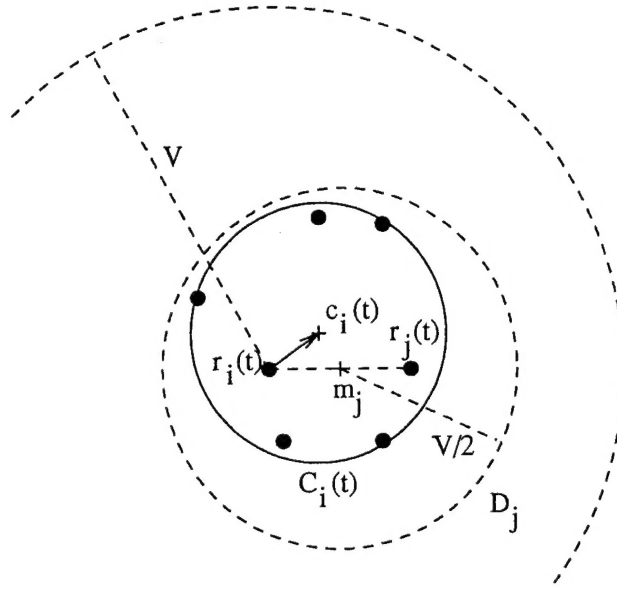
Figure 5: Illustration for the proof of Lemma 3.

Proof Fix a robot $r_i \in S$. Since the diameter of $CH(t)$ is no greater than V, all the robots in S are visible from r_i at t (and of course, no other robots are visible from r_i). Thus $S_i(t) = S$, and hence the center $c_i(t)$ of $C_i(t)$ towards which r_i moves is identical for all robots in S. Now, let r_j be an arbitrary robot in $S_i(t)$. Since both r_i and r_j are inside $C_i(t)$, the midpoint m_j of $r_i(t)$ and $r_j(t)$ is within distance $V/2$ of $c_i(t)$. See Figure 5. So $c_i(t)$ is inside the disc D_j with center m_j and radius $V/2$, and hence $GOAL \leq \ell_j$, where $GOAL$ and ℓ_j are defined in Section 3. Since this is true for any $r_j \in S_i(t)$, we have $GOAL \leq LIMIT$ for r_i. So $MOVE = GOAL = dist(r_i(t), c_i(t))$ for r_i, that is, r_i moves to $c_i(t)$. $\square$

Therefore, what remains to be proved is that the diameter of $CH(t)$ decreases to a value that is not greater than V. (Note that Lemma 2 alone does not guarantee this.) Now, by Lemma 2, we know at least that the series $\{CH(t) : t = t_0, t_0 + 1, \ldots\}$ converges. So suppose that it converges to CH, where CH must clearly be a convex polygon, including, as special cases, a point and a line segment. We will show in Lemma 5 given below that CH is indeed a single point. We need the following technical lemma, Lemma 4, in order to prove Lemma 5.

Lemma 4 *Suppose that at t, (1) the robots that are visible from r_j are located on the arc or the apex of a sector with apex $r_j(t)$, apex angle φ and radius V, where $0 \leq \varphi < \pi$, and (2) at least one robot that is visible from r_j is located on the arc of this sector. (See Figures 6 and 7.) Then at t, r_j moves over distance at least $\min\{V/2, V\cos(\varphi/2)\}$ and at most $V/\sqrt{2}$.*

Proof Let φ', $0 \leq \varphi' \leq \varphi$, be the smallest angle such that the robots visible from r_j lie inside the wedge with apex $r_j(t)$ and apex angle φ'. The lemma follows from the following argument.
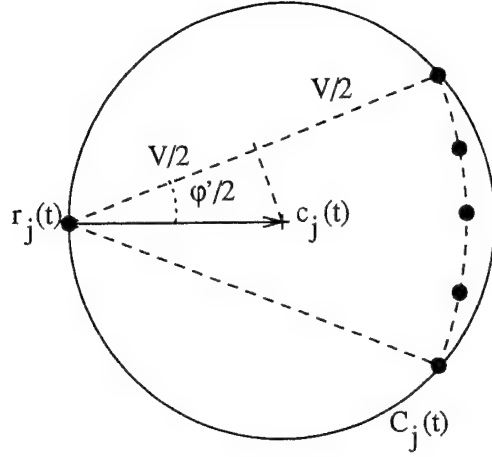Case 1: $0 \leq \varphi' \leq \pi/2$.
See Figure 6. In this case,

Figure 6: The case $0 \leq \varphi' \leq \pi/2$.

$$GOAL = (V/2)/\cos(\varphi'/2)$$

and

$$LIMIT \geq V/2.$$

Then, since $MOVE = \min\{GOAL, LIMIT\}$ and

$$V/2 \leq (V/2)/\cos(\varphi'/2) \leq V/\sqrt{2},$$

we have

$$V/2 \leq MOVE \leq V/\sqrt{2}.$$

Case 2: $\pi/2 < \varphi' < \pi$.
See Figure 7. In this case,

$$GOAL = LIMIT = V\cos(\varphi'/2),$$

and hence

$$MOVE = V\cos(\varphi'/2).$$

Thus

$$V\cos(\varphi/2) \leq MOVE < V/\sqrt{2}.$$

□

Lemma 5 *CH is a point.*

Proof First, we assume that CH is a convex polygon other than a single point or a line segment, and derive a contradiction. Let α be an arbitrary corner of CH, and φ the internal angle at α. Let $\delta > 0$ be an arbitrary (small) real number. By the assumption of convergence, there exists a sufficiently large time instant t_1 ($\geq t_0$) such that at any $t \geq t_1$, all the robots in S are in the δ-neighborhood of CH, and there exists at least one robot in the δ-neighborhood of α. Let CH' be the convex polygon obtained from CH by translating
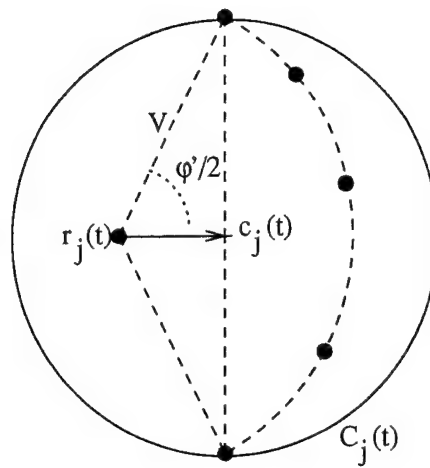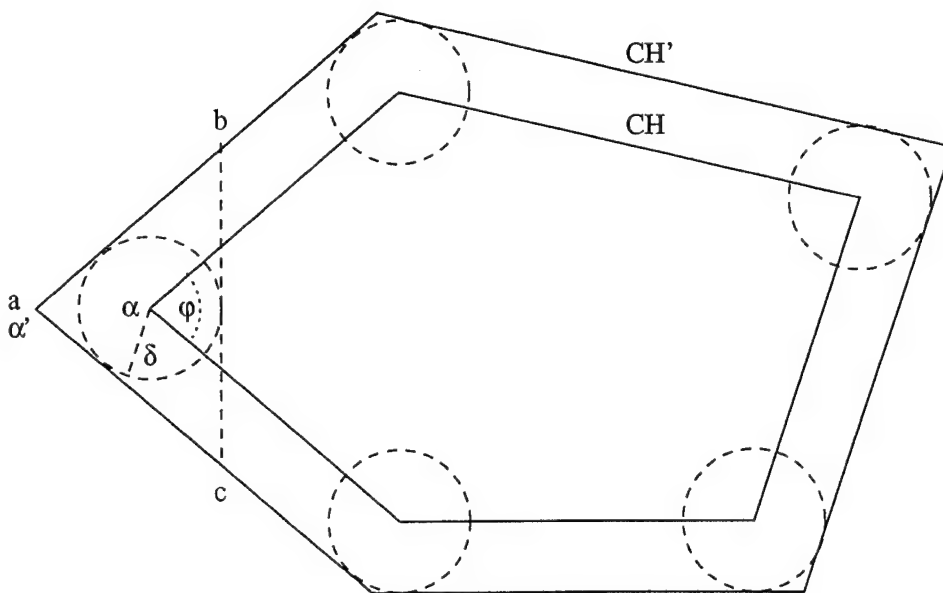
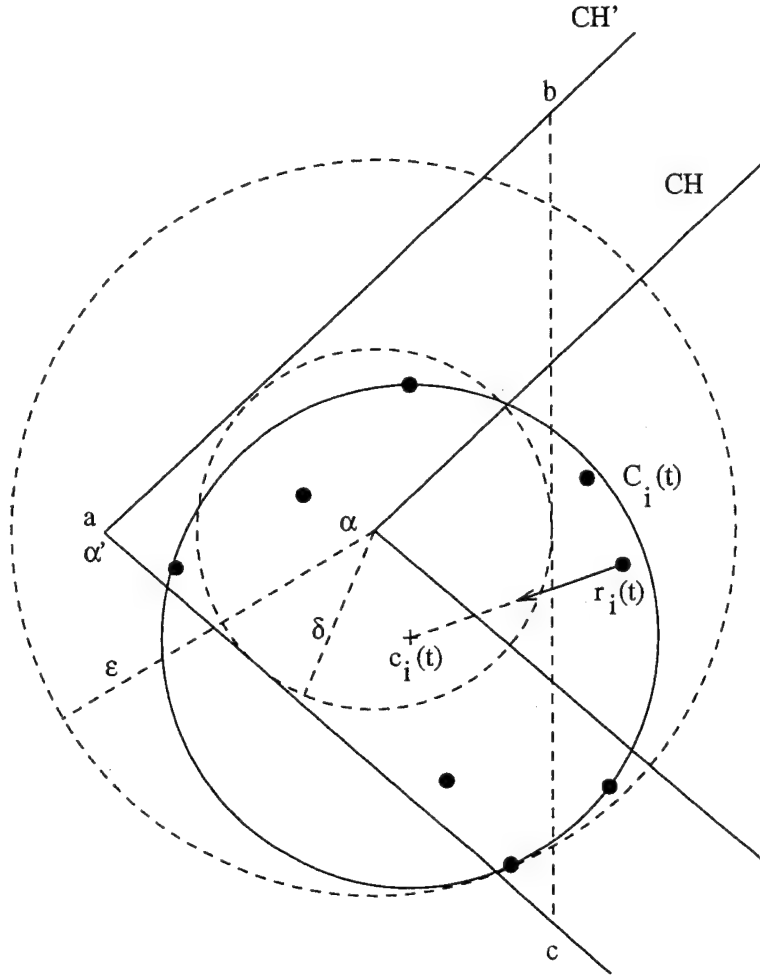Figure 7: The case $\pi/2 < \varphi' < \pi$.

Figure 8: CH and CH'.

Figure 9: Illustration for Case 1 of the proof of Lemma 5.

each edge of CH outward over distance δ. See Figure 8. Note that CH' contains the δ-neighborhood of CH. Let α' be the corner of CH' corresponding to α. Let $\triangle abc \subseteq CH'$ be the smallest isosceles triangle containing the δ-neighborhood of α, such that $ab = ac$ and corner a is at α'. Then there exists at least one robot in $\triangle abc$ at any time instant after t_1. So we let r_i be a robot that is in $\triangle abc$ at $t + 1$, that is, $r_i(t+1) \in \triangle abc$, where $t > t_1$, and examine the position $r_i(t)$ of r_i at t. We use symbols $S_i(t)$, $C_i(t)$ and $c_i(t)$ defined previously. By Proposition 1 and the fact that all the robots in S are in CH' at t, the center $c_i(t)$ of $C_i(t)$ is in CH'. There are two cases, depending on the relative positions of $c_i(t)$ and $\triangle abc$.

Case 1: $c_i(t)$ is inside $\triangle abc$.

See Figure 9. By Proposition 1 and the fact that all the robots in S are in CH' at t, there exist two points p, q in CH' such that $\overline{pq}$ is a diameter of $C_i(t)$. This, together with the condition that $c_i(t) \in \triangle abc$, implies that (1) there exists some $\epsilon > 0$ that depends only on δ and CH, such that all the robots in $S_i(t)$ (including $r_i(t)$) are in the ϵ-neighborhood of α,
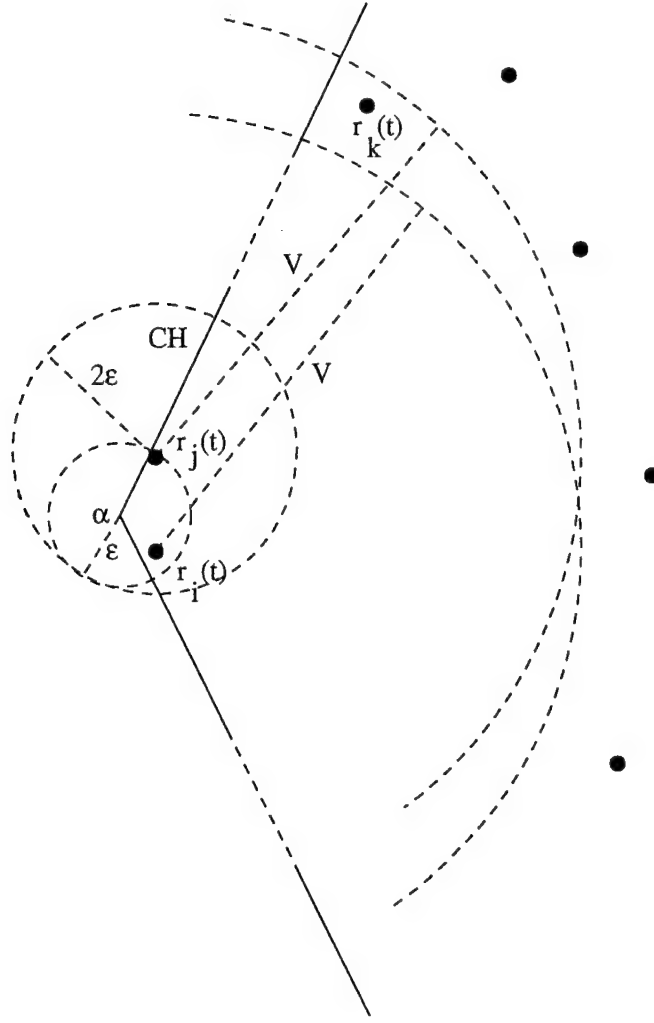
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Figure 10: r_i, r_j and r_k.

and (2) this ϵ can be made arbitrarily close to 0 by choosing sufficiently small δ. So assume that ϵ is very small. Then, since the robots in S constitute a connected component of G_t and ϵ is much smaller than the distance from α to any other corner of CH, there is at least one robot, say r_k, that is not visible from r_i at t, but that is visible from some robot, say r_j visible from r_i. That is, there exist $r_j \in S_i(t)$ and $r_k \in S - S_i(t)$ such that $r_k \in S_j(t)$. See Figure 10. This means that the robots that are visible from r_j at t are either within distance 2ϵ (≈ 0) of $r_j(t)$, or at distance greater than $V - 2\epsilon$ ($\approx V$) of $r_j(t)$ (and there is at least one such robot, called r_k above). So the situation is similar to that described in Lemma 4, and thus the distance of the movement of r_j at t must be almost the same as that given in Lemma 4. Thus at $t+1$, r_j is at distance at least about $\min\{V/2, V\cos(\varphi/2)\}$ from α, and at distance at most about $V/\sqrt{2}$ from α, where δ can be chosen in advance so that ϵ is much smaller than $\min\{V/2, V\cos(\varphi/2)\}$. Thus at $t+1$, r_j is visible from every robot in the ϵ-neighborhood of α. So if δ (and thus ϵ) is chosen sufficiently small, then at $t+1$,
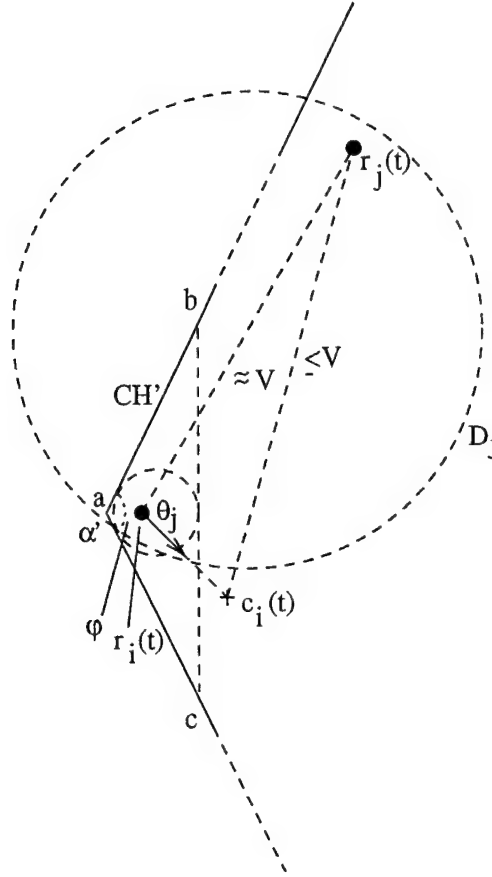
Figure 11: r_i and r_j.

every robot in the ϵ-neighborhood of α moves out of that region. So immediately before the move at $t+2$, there are no robots in the δ-neighborhood of α. This is a contradiction. (End of Case 1)

Case 2: $c_i(t)$ is outside $\triangle abc$.

See Figure 11. Since $r_i(t) \notin \triangle abc$ implies $r_i(t+1) \notin \triangle abc$ contradicting the assumption, we have $r_i(t) \in \triangle abc$. Also, clearly the distance over which r_i moves at t is $MOVE = LIMIT$, where $LIMIT$ is not greater than the length of a longest side of $\triangle abc$, which can become arbitrarily small if δ is chosen to be small, for any fixed value of φ. Now, by the definition of $LIMIT$, in order for the value of $LIMIT$ to be small, there must exist a robot $r_j \in S_i(t)$ such that $dist(r_i(t), r_j(t)) \approx V$ and $\theta_j = \angle c_i(t) r_i(t) r_j(t)$ is close to or greater than $\pi/2$. On the other hand, since $c_i(t)$ is the center of the smallest enclosing circle of $S_i(t)$, $dist(r_j(t), c_i(t)) \leq V$ holds. Therefore, if δ is chosen to be sufficiently small, then $dist(r_i(t), c_i(t))$ can become arbitrarily close to 0. So, if the value of δ is modified to be slightly larger (but still sufficiently small) so that $c_i(t)$ is inside (new) $\triangle abc$, then the argument used in Case 1 can be applied to show that the diameter of $C_i(t)$ must be very small. This implies that $dist(r_i(t), r_j(t))$ must be very small, contradicting $dist(r_i(t), r_j(t)) \approx V$. (End of Case 2)

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The claim that CH is not a line segment can be shown in a similar way, and we omit the details. Therefore, eventually the diameter of $CH(t)$ becomes no greater than V, and then by Lemma 3, all the robots in S move to a single point in one step. Thus CH is a point. □

By the lemmas given above, we obtain the following theorem.

Theorem 1 *The algorithm solves the single point formation problem correctly.* □

We remark that the proof of the correctness of the algorithm of Section 3 is much more complex than that of an algorithm given in [14] for converging the robots with unlimited visibility to a single point. This is due to the fact that, under limited visibility, the behavior of a robot is based only on local information available to that robot, whereas the correctness of the algorithm can only be derived from the global behavior of the entire set of robots.

5 Agreement Problems

In this section, we discuss two basic agreement problems for the robots, namely, agreement on a common x-y coordinate system and agreement on the initial distribution. Here, agreement means that the robots should obtain, in finite steps, a common understanding of the given concept. As we discussed in Section 2, however, under limited visibility some robots may never belong to the same connected component of G_t for any t during the execution of the given algorithm. So we cannot expect *all* robots to agree on the given concept. So in the following, we only require the robots that belong to the same connected component of G_0 to reach an agreement. (Of course, additional robots that happen to be merged into a new connected component may also be able to agree.)

5.1 Agreement on an x-y coordinate system

Agreement on a common x-y coordinate system means that the robots should obtain, in finite steps, a common understanding of the origin, unit distance, and direction of the positive x-axis. As is shown in [14], however, agreement on direction is not possible in general, even if the robots have unlimited visibility.

On the other hand, agreement on the origin and unit distance can be achieved using the algorithm of Section 3 for forming a point. As we discussed in Section 4, the robots that belong to the same connected component of G_0 (and possibly some additional robots) eventually move to the same point, say p, at some time instant t, in such a way that at this moment, they do not see any other robot not located at p. At this moment, the robots can agree to use p as the common origin. Next, at $t+1$, each robot r_i in S moves to the midpoint of p and its previous position $r_i(t-1)$. Since the distance between p and $r_i(t-1)$ is at most V by the definition of $LIMIT$, the distance between p and $r_i(t+1)$ is at most $V/2$, and thus any two robots in S are still mutually visible at $t+1$. Then the robots can adopt, as the common unit distance, the radius of the smallest enclosing circle of the positions of the robots in S at $t+1$. Note that by construction, the size of the unit distance is no more than $V/2$.

The operation described above works correctly, except when additional robots not in S become visible to some robots in S at $t+1$. If this happens, then the new set of robots

(including the robots in S) that constitute a new connected component of G_{t+1} must repeat the entire process, starting with the agreement on the origin. (This is unavoidable in general, since there can be more than one connected component in G_0.) To see if this has happened, we let the robots in S execute *one step* of the algorithm of Section 3 at $t+1$. If no additional robots become visible at $t+1$ to any of the robots in S, then since the diameter of the convex hull of the positions of the robots in S at $t+1$ is not greater than V, by Lemma 3 all the robots in S will again move to a single point, say p', at $t+1$. (p' is not necessarily the same as p.) If on the other hand additional robots become visible at $t+1$ to some robots in S, then either (1) not all the robots in S move to the same point at $t+1$, or (2) all robots in S move to the same point at $t+1$ and all of them find at $t+2$ that the number of robots in their connected component has increased. (Note that by Lemma 1, robots that are mutually visible remain mutually visible during the execution of the algorithm of Section 3.) In either case, the robots in S realize that they have to restart the process for agreement on the origin and unit distance.

The following theorem follows from the discussion given above. We omit the proof.

Theorem 2 *The agreement problem on the origin and unit distance is solvable for synchronous robots under limited visibility.* $\square$

5.2 Agreement on the initial distribution

Agreement on the initial distribution requires that the robots in a connected component of G_0 obtain a correct understanding of the initial positions of all the robots in that component. This can be solved as follows.

First, the robots agree on the origin and unit distance, using the method given in the previous subsection. Let p be the origin, and d the size of the unit distance, where $d \leq V/2$ by construction. Then all the robots move to p, say at t. At $t+1$, each robot r_i in S moves towards its initial position $r_i(0)$, over distance $(1 - 1/2^x)d$, where x is the distance from p to $r_i(0)$ measured in the units of d. Note that since $0 \leq x < \infty$, we have $0 \leq 1 - 1/2^x < 1$, and hence $0 \leq (1 - 1/2^x)d < d \leq V/2$. Thus at $t+1$, the robots in S are still mutually visible, and every robot r_i in S can figure out, for every robot r_j in S, the direction of $r_j(0)$ from p and distance to $r_j(0)$ from p, by observing the position $r_j(t+1)$ and using the knowledge of the size of d. Therefore at $t+1$, the robots in S have discovered and agreed on their initial distribution. The case when additional robots become visible to the robots in S during this operation can be handled easily, as we did in the previous subsection. We omit the details.

The following theorem follows from the discussion given above. We omit the proof.

Theorem 3 *The agreement problem on the initial distribution is solvable for synchronous robots under limited visibility.* $\square$

6 Conclusion

We discussed formation and agreement problems for autonomous, synchronous robots with limited visibility. The algorithm we presented for the single point formation problem is *oblivious*, in the sense that the position of robot r_i at time $t+1$ is determined only from the positions of other robots that r_i observes at t. One might wonder whether the same problem

can still be solved by an oblivious algorithm when the robots are *asynchronous* (i.e., when the robots are not guaranteed to move simultaneously all the time), but unfortunately, it has been shown in [14] that no oblivious algorithm exists for the single point formation problem for asynchronous robots, even if the robots have unlimited visibility. On the other hand, a *nonoblivious* algorithm for the single point formation problem has been reported for asynchronous robots with unlimited visibility in [14]. It is an interesting open problem to determine whether or not the same problem can be solved by a nonoblivious algorithm for asynchronous robots with limited visibility. Furthermore, it is not known exactly what class of geometric figures can be formed by synchronous robots under limited visibility. (A point is an example of such a figure.) Some results in this direction have been reported in [14] for synchronous robots with unlimited visibility. Investigation of this problem for robots with limited visibility is suggested for future research.

As for the agreement problems we discussed, our results show that the limitation on the visibility of the robots has no effect on whether or not they are solvable, with, of course, a minor qualification that under limited visibility, not all the robots may be able to agree on the given concept. However, under limited visibility, the robots must first get sufficiently close to each other (for example, they move to a single point), before reaching an agreement. In many cases, this is not necessary if the robots have unlimited visibility. So the limitation on the visibility tends to increase the complexity of the algorithms. One challenging open problem regarding agreement is to decide whether or not *asynchronous* robots with limited visibility can discover and agree on their initial distribution. We suggest this problem for future research.

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